MA380 – Computational Algebraic Geometry **Spring 2018**
TTh 2:00 p.m.–3:15 p.m. (M-1-207)
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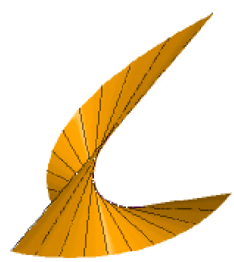
Course Description

An *affine algebraic variety* is the solution set of a system of polynomial equations. Low-dimensional examples include ellipses, hyperbolas, and parabolas, and also the surfaces shown below:

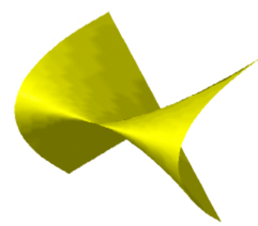
$x^2 + y^2 - x^2z + y^2z + z^2 = 1$

Cayley's surface, with four *double points* and *tetrahedral symmetry.*



$3x^2y^2 + 6xyz = 4x^3z + 4y^3 + z^2$

Tangent surface to the twisted cubic, an example of a *ruled surface* with a crease along its base curve.



$x^2 = y^2z$

Whitney's umbrella, showing a *pinch point singularity.*

This course is an introduction to the geometry of affine algebraic varieties, with emphasis on two core themes:

The Algebra-Geometry Dictionary: Descartes' introduction of *coordinates* in 1637 hinted at the essential unity of algebra and geometry. Three and a half centuries of subsequent development showed the correspondence to be deeper and more complete than Descartes could have foreseen. We will describe a "dictionary" by which one can translate geometric ideas into algebraic ones, and vice-versa.

Buchberger's Algorithm: In linear algebra one has the familiar *Gauss-Jordan algorithm* for solving systems of linear equations, and by extending this algorithm in various ways one can compute (almost) everything worth knowing about the solutions of such systems. *Buchberger's algorithm*, first described in the 1960s, generalizes Gauss-Jordan elimination to systems of polynomial equations of arbitrary degree, and various extensions of it compute almost everything worth knowing about affine algebraic varieties.[1] Because Buchberger's algorithm is computationally intensive, one almost never carries it out by hand; instead, one makes use of **computer algebra systems** for computation and visualization.

Students who complete this course will acquire a strong foundation for the study of various applications both within and outside mathematics. In particular, our textbook contains material on **robotics**, **computer aided design**, **automatic theorem proving**, **invariant theory**, **projective geometry**, and **computer vision**.

Prerequisites

Admission to the course is contingent upon successful completion of MA260 or an equivalent linear algebra course.

Text

There is one required text for the course: *Ideals, Varieties, and Algorithms*, fourth edition, by David Cox, John Little, and Donal O'Shea.

Grading

Course grades are based on weekly quizzes (20%), two in-class tests (20% each), and a cumulative final exam (40%).

Reading and class preparation

There is a reading assignment associated with each class period. Although it is not generally possible to discuss every topic in class, students are responsible for the entire content of the reading assignment. *Test and exam questions may cover reading material not discussed explicitly in class.* Consequently it is very important to complete the reading assignments on time and to come to class prepared with questions.

Make-up tests

Tests may be rescheduled only in cases of serious illness, bereavement, or other circumstances of similar gravity. Whenever possible, arrangements for make-up tests must be made *in advance* of the regularly scheduled testing time.

Accomodations for students with disabilities

Section 504 of the Americans with Disabilities Act of 1990 offers guidelines for curriculum modifications and adaptations for students with documented disabilities. If applicable, students may obtain adaptation recommendations from

[1] At least in principle. Buchberger's algorithm has high computational complexity, and it is easy to make examples which overwhelm even the strongest computers.

the Ross Center for Disability Services, CC-UL-211, (617-287-7430). The student must present these recommendations and discuss them with each professor within a reasonable period, preferably by the end of the Drop/Add period.

Student conduct

Students are required to adhere to the University Policy on Academic Standards and Cheating, to the University Statement on Plagiarism and the Documentation of Written Work, and to the Code of Student Conduct. The Code is available online at the following web site:

`https://www.umb.edu/editor_uploads/images/life_on_campus/Code_of_Conduct_5-14-14.pdf`

Please pay particular attention to Section XII, paragraphs 1 and 5. In this course, you will be permitted to use a short note sheet during exams, provided that you have prepared the sheet yourself. Your exam responses may quote your lecture notes or the course textbook without attribution, but material taken from any other source must be properly attributed to its author. In addition, the use of electronic devices during exams is *expressly prohibited*. Violation of these policies will result in disciplinary action.

Web page

This syllabus and other course materials are available on-line at

`http://cartan.math.umb.edu/wiki/index.php/Math_380,_Spring_2018`

Schedule of topics

Week 1: Introduction. Polynomials and affine space.

Week 2: Affine varieties. Parametrizations.

Week 3: Ideals. Polynomials in one variable.

Week 4: Introduction to Gröbner bases. Monomial orderings.

Week 5: The division algorithm. Monomial Ideals and Dickson's Lemma.

Week 6: The Hilbert basis theorem. First midterm (Thursday, March 1; covers assignments 1–4).

Week 7: Gröbner bases. Buchberger's algorithm.

Week 8: First applications of Gröbner bases. The elimination and extension theorems.

Week 9: The geometry of elimination. Implicitization.

Week 10: Singular points and envelopes. Gröbner bases and the extension theorem.

Week 11: The Nullstellensatz. Second midterm (Thursday, April 12; covers assignments 5–9).

Week 12: Radical ideals and the ideal-variety correspondence. Sums, products, and intersections of ideals.

Week 13: Zariski closures, quotients, and saturations. Irreducible varieties and prime ideals.

Week 14: Decomposition of a variety into irreducibles. Primary decomposition.

Week 15: Epilogue: the algebra-geometry dictionary as an abstract Galois correspondence.