MATH 141 Spring 2016 Final exam practice problems

1. Find the following limits, or show that they do not exist:

   (a) $\lim_{x\to 0} \dfrac{x^2}{1-\cos x}$

   (b) $\lim_{x\to \infty} x\tan(3/x)$

   (c) $\lim_{x\to \infty} \left(1+\dfrac{5}{x}\right)^x$

   (d) $\lim_{x\to \infty} \dfrac{e^{3x}-x-1}{e^{2x}-x-1}$

   (e) $\lim_{x\to \infty} \ln(x+1)-\ln(x)$

   (f) $\lim_{x\to 0^+} x^2\ln x$

2. Evaluate the following integrals.

   (a) $\displaystyle\int \sin^5 x\, dx$

   (b) $\displaystyle\int \frac{1}{\sqrt{7-6x-x^2}}\, dx$

   (c) $\displaystyle\int \frac{5x^2+3x+7}{(x+1)(x^2+2)}\, dx$

   (d) $\displaystyle\int xe^{3x}\, dx$

   (e) $\displaystyle\int \frac{dt}{t^2\sqrt{t^2-16}}$

   (f) $\displaystyle\int \frac{x^2+1}{(x-3)(x-2)^2}\, dx$

3. Evaluate the following integrals, or show that they diverge.

   (a) $\displaystyle\int_1^\infty \frac{x^2}{x^3+1}\, dx$

   (b) $\displaystyle\int_2^\infty \frac{1}{x^2+4}\, dx$

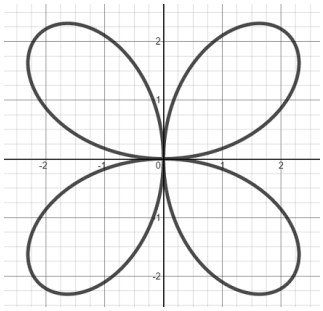
   (c) $\displaystyle\int_{-1}^8 \frac{1}{x^{1/3}}\, dx$

   (d) $\displaystyle\int_0^\pi \tan\theta\, d\theta$

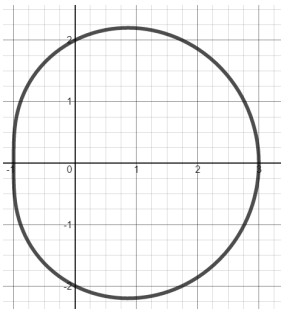
4. Find the length of the curve $x = 3e^{2t}$, $y = 2e^{3t}$, with $0 \le t \le 1$.

5. Find the length of the curve $x = \sin t + \cos t$, $y = \sin t - \cos t$, with $0 \le t \le \pi$.

6. Find the area of one petal of $r = 3\sin 2\theta$:



7. Find the area enclosed by the graph of $r = 2 + \cos\theta$:



8. The series $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+2} + 3^n}{5^{2n}}$ is convergent. Find its sum.

9. The series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{4^n}$ is convergent. Find its sum.

10. The series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3} - \frac{1}{(n+2)^3} \right)$ is convergent. Find its sum.

11. The series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}} \right)$ is convergent. Find its sum.

12. For each of the following series, determine whether it converges or diverges. Explain your reasoning.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+5}}$

    (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+4}{10n^2+1}$

    (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(n+1)!}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3-2}$

13. Determine the interval of convergence of the following power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{2n+1}$

14. (a) Express $\int x^3 \cos(x^2)\, dx$ as a power series. Write out the first 4 nonzero terms, and then write the series using proper sigma notation.

15. (a) Express $\int \frac{e^x-1}{x}\, dx$ as a power series. Write out the first 4 nonzero terms, and then write the series using proper sigma notation.