Useful formulas

1. (Arc length) Length of the curve $y = f(x)$ from $x = a$ to $x = b$: $\displaystyle\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$.

2. (Surface area) Area of the surface of revolution obtained by rotating $y = f(x)$ from $x = a$ to $x = b$ about the x-axis (assuming $y \geq 0$): $\displaystyle 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$.

3. (Surface area) Area of the surface of revolution obtained by rotating $y = f(x)$ from $x = a$ to $x = b$ about the y-axis: $\displaystyle 2\pi \int_a^b x\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$.

4. (Slope of a parametric curve) $\displaystyle\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

5. (Second derivative of a parametric curve) $\displaystyle\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$.

6. (Parametric arc length) Length of $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ between $t = a$ and $t = b$: $\displaystyle\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$.

7. (Parametric area) Area between a parametric curve and the x-axis: $\displaystyle\int_a^b y\frac{dx}{dt}\, dt$. (This formula is valid when y is positive and x is increasing with t, or when y is negative and x is decreasing. In the remaining two cases, add a minus sign to the formula.)

8. (Relations between rectangular and polar coordinates) If the same point is represented by rectangular coordinates (x, y) and by polar coordinates (r, θ), then $x = r\cos(\theta)$, $y = r\sin(\theta)$, $r^2 = x^2 + y^2$, and $\tan(\theta) = \frac{y}{x}$.

9. (Non-uniqueness of polar coordinates) If the same point is represented by two sets of polar coordinates (r_1, θ_1) and (r_2, θ_2), then either (i) $r_1 = r_2$ and $\theta_1 - \theta_2$ is an even multiple of π, or (ii) $r_1 = -r_2$ and $\theta_1 - \theta_2$ is an odd multiple of π, or (iii) $r_1 = r_2 = 0$.

10. (Polar arc length) The length of the curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$ is $\displaystyle\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$.

11. (Polar area) The area enclosed by the curve $r = f(\theta)$ and the rays $\theta = a$ and $\theta = b$ is $\displaystyle\frac{1}{2}\int_a^b r^2\, d\theta$.

12. (Useful trigonometric identities) $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ and $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$. (Note that the usual double-angle identities can be obtained from these by putting $\alpha = \beta = \theta$).