MA480 – Introduction to Computational Algebraic Geometry II

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Course Description

This course is a continuation of Math 380, *Introduction to computational algebraic geometry*. Topics include regular and rational functions, invariant theory, projective geometry, and dimension theory. The course concludes with more modern developments in algebraic geometry: categories, the Spec and Proj functors, and an introduction to sheaves and schemes.

Prerequisites

Admission to the course is contingent upon successful completion of MA380 (equivalent to the instance of MA480 offered in the spring of 2013).

Text

There is one required text for the course: *Ideals, Varieties, and Algorithms*, third edition, by David Cox, John Little, and Donal O'Shea.

Grading

Course grades are based on weekly quizzes (20%), two in-class tests (20% each), and a cumulative final exam (40%).

Reading and class preparation

There is a reading assignment associated with each class period. Although it is not generally possible to discuss every topic in class, students are responsible for the entire content of the reading assignment. *Test and exam questions may cover reading material not discussed explicitly in class.* Consequently it is very important to complete the reading assignments on time and to come to class prepared with questions.

Make-up tests

Tests may be rescheduled only in cases of serious illness, bereavement, or other circumstances of similar gravity. Whenever possible, arrangements for make-up tests must be made *in advance* of the regularly scheduled testing time.

Student conduct

Students are required to adhere to the University Policy on Academic Standards and Cheating, to the University Statement on Plagiarism and the Documentation of Written Work, and to the Code of Student Conduct as delineated in the catalog of Undergraduate Programs, pp. 44–45 and 48–52. The Code is available online at the following web site:

http://www.umb.edu/editor_uploads/images/life_on_campus/CSC.pdf

Web page

This syllabus and other course materials are available on-line at

http://cartan.math.umb.edu/wiki/index.php/Math_480,_Spring_2014

Course Calendar

Homework problems should be done prior to the due date but are not to be handed in. One problem from each assignment will appear on the weekly quiz.

Monday, January 27: Introduction.

Wednesday, January 29: Review of the first semester's results.

Friday, January 31: Polynomial mappings.

Read before class: Section 5.1.

Monday, February 3: Quotients of polynomial mappings.

Read before class: Section 5.2.

Do before class: Assignment 1.

Wednesday, February 5: Algorithmic computation in quotients of polynomial rings.

Read before class: Section 5.3.

Friday, February 7: More on algorithmic computation.

Monday, February 10: The coordinate ring of an affine variety.

Read before class: Section 5.4.

Do before class: Assignment 2.

Wednesday, February 12: The field of rational functions.

Read before class: Section 5.5.

Friday, February 14: More on rational functions.

Wednesday, February 19: Symmetric polynomials.

Read before class: Section 7.1.

Do before class: Assignment 3.

Friday, February 21: More on symmetric polynomials.

Monday, February 24: Finite matrix groups and rings of invariants.

Read before class: Section 7.2.

Do before class: Assignment 4.

Wednesday, February 26: Generators for the ring of invariants. Read before class: Section 7.3.

Friday, February 28: More on generating invariants.

Monday, March 3: Relations among generators and the geometry of orbits.

Read before class: Section 7.4.

Do before class: Assignment 5.

Wednesday, March 5: The projective plane.

Read before class: Section 8.1.

Friday, March 7: Projective space and projective varieties.

Read before class: Section 8.2.

Monday, March 10: Exam 1, sections 5.1, 5.2, 5.3, 5.4, 5.5, 7.1, 7.2, 7.3, and 7.4.

Do before class: Assignment 6.

Wednesday, March 12: The projective algebra-geometry dictionary.

Read before class: Section 8.3.

Friday, March 14: More on the dictionary.

Monday, March 24: The projective closure of an affine variety.

Read before class: Section 8.4.

Do before class: Assignment 7.

Wednesday, March 26: Projective elimination theory.

Read before class: Section 8.5.

Friday, March 28: More on elimination theory.

Monday, March 31: Quadric hypersurfaces.

Read before class: Section 8.6.

Do before class: Assignment 8.

Wednesday, April 2: Bezout's Theorem.

Read before class: Section 8.7.

Friday, April 4: More on Bezout's Theorem.

Monday, April 7: The variety of a monomial ideal.

Read before class: Section 9.1.

Do before class: Assignment 9.

Wednesday, April 9: The complement of a monomial ideal.

Read before class: Section 9.2.

Friday, April 11: More on monomial ideals.

Monday, April 14: The Hilbert function and the dimension of a variety.

Read before class: Section 9.3.

Do before class: Assignment 10.

Wednesday, April 16: Exam 2, sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, and 8.7.

Friday, April 18: More on the Hilbert function.

Wednesday, April 23: Elementary properties of dimension.

Read before class: Section 9.4.

Do before class: Assignment 11.

Friday, April 25: Dimension and algebraic independence.

Read before class: Section 9.5.

Monday, April 28: Dimension and non-singularity.

Read before class: Section 9.6.

Do before class: Assignment 12.

Wednesday, April 30: The tangent cone. Read before class: Section 9.7. Friday, May 2: More on the tangent cone.

Read before class: Section 9.8.

Monday, May 5: What is a variety? Do before class: Assignment 13.

Wednesday, May 7: Sheaves of rings.

Friday, May 9: Spec and Proj.

Monday, May 12: Categories. The algebra-geometry dictionary as an equivalence of categories.

Do before class: Assignment 14.

Wednesday, May 14: Schemes.

Do before class: Assignment 15.