Fall 2016

MA480 – Information Theory TTh 2:00 p.m.–3:15 p.m. (Y-2-2310)

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Course Description

Math 480 is an elementary introduction to Information Theory, emphasizing its original applications: reliable communication over noisy channels, and data compression. The course begins with a brief review of finite probability theory. We then define the fundamental quantities of Information Theory, information and entropy, and investigate their essential properties. After this, we develop the basics of error correcting codes, leading to Shannon's Noisy Channel Theorem, and the fundamental ideas of data compression, leading to the Lossless Compression Bound. If time permits, we may also discuss more recent applications of the theory, such as statistical physics, evolutionary biology, and machine learning.

Prerequisites

Students should have successfully completed Math 260, or an equivalent linear algebra course.

Text

There is one required text for the course: *Introduction to Information Theory* and Data Compression, second edition, by Darrel Hankerson, Greg A. Harris, and Peter D. Johnson, Jr.

Grading

Course grades are based on weekly quizzes (20%), two in-class tests (20% each), and a cumulative final exam (40%).

Reading and class preparation

There is a reading assignment associated with each class period. Although it is not generally possible to discuss every topic in class, students are responsible for the entire content of the reading assignment. *Test and exam questions may cover reading material not discussed explicitly in class.* Consequently it is very important to complete the reading assignments on time and to come to class prepared with questions.

Make-up tests

Tests may be rescheduled only in cases of serious illness, bereavement, or other circumstances of similar gravity. Whenever possible, arrangements for make-up tests must be made *in advance* of the regularly scheduled testing time.

Student conduct

Students are required to adhere to the University Policy on Academic Standards and Cheating, to the University Statement on Plagiarism and the Documentation of Written Work, and to the Code of Student Conduct. The Code is available online at the following web site:

https://www.umb.edu/editor_uploads/images/life_on_campus/Code_of_Conduct_5-14-14.pdf

Please pay particular attention to Section XII, paragraphs 1 and 5. In this course, you will be permitted to use a short note sheet during exams, provided that you have prepared the sheet yourself. Your exam responses may quote your lecture notes or the course textbook without attribution, but material taken from any other source must be properly attributed to its author. In addition, the use of electronic devices during exams is *expressly prohibited*. Violation of these policies will result in disciplinary action.

Web page

This syllabus and other course materials are available online at

http://cartan.math.umb.edu/wiki/index.php/Math_480,_Fall_2016

Course Calendar

Homework problems should be done prior to the due date but are not to be handed in. One problem from each assignment will appear on the weekly quiz.

Tuesday, September 6: Introduction.

Thursday, September 8: Finite probability spaces and random variables.

Read before class: Sections 1.1 and 1.2.

Tuesday, September 13: Conditional probability and independence.

Read before class: Sections 1.3 and 1.4.

Do before class: Assignment 1.

Thursday, September 15: Bernoulli trials and counting.

Read before class: Sections 1.5 and 1.6.

Tuesday, September 20: Expectation value and the Law of Large Numbers. Read before class: Sections 1.8 and 1.9.

Do before class: Assignment 2.

Thursday, September 22: Information.

Read before class: Section 2.1.

Tuesday, September 27: Systems of events and mutual information.

Read before class: Section 2.2.

Do before class: Assignment 3.

Thursday, September 29: Entropy. Read before class: Sections 2.3 and 2.4.

Tuesday, October 4: Channels and transition probabilities.Read before class: Sections 3.1 and 3.2.Do before class: Assignment 4.

Thursday, October 6: Input frequencies and channel capacity. Read before class: Sections 3.3 and 3.4.

Tuesday, October 11: Prefix-free codes and the Kraft-McMillan inequality.Read before class: Sections 4.1 and 4.2.Do before class: Assignment 5.

Thursday, October 13: Average word length and Huffman's algorithm. Read before class: Section 4.3 (excluding 4.3.1).

Tuesday, October 18: Exam 1 (covers assignments 1–5). Do before class: Assignment 6

Thursday, October 20: Validity of Huffman's algorithm. Read before class: Section 4.3.1.

Tuesday, October 25: Input frequency optimization.

Read before class: Section 4.4.

Do before class: Assignment 7.

Thursday, October 27: Error correction.

Read before class: Section 4.5.

Tuesday, November 1: More on error correction.

Do before class: Assignment 8.

Thursday, November 3: Shannon's Noisy Channel Theorem.

Read before class: Section 4.6.

Tuesday, November 8: Binary symmetric channel with equal source frequencies; the information rate of a code.

Read before class: Sections 4.7 and 4.8.

Do before class: Assignment 9.

Thursday, November 10: Replacement via encoding scheme; review of the prefix condition.

Read before class: Sections 5.1 and 5.2.

Tuesday, November 15: Exam 2 (covers assignments 6–9).

Do before class: Assignment 10.

Thursday, November 17: Choosing an encoding scheme.

Read before class: Section 5.3.

Tuesday, November 22: Shannon's lossless compression bound.

Read before class: Section 5.4.

Do before class: Assignment 11.

Tuesday, November 29: Zeroth-order arithmetic coding.

Read before class: Section 6.1 (excluding 6.1.1 and 6.1.2).

Do before class: Assignment 12.

Thursday, December 1: More on zeroth-order arithmetic coding.

Read before class: Sections 6.1.1 and 6.1.2.

Tuesday, December 6: Advantages of arithmetic coding.

Read before class: Section 6.2.

Do before class: Assignment 13.

Thursday, December 8: Disadvantages of arithmetic coding.

Read before class: Section 6.3.

Tuesday, December 13: Review. Do before class: Assignment 14.